# **Theoretical Review of CP Violation**

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### **Abstract**

The focus of our considerations is the B-meson system, which will provide stringent tests of the Kobayashi–Maskawa mechanism of CP violation in this decade. After a classification of the main possible strategies to achieve this goal, we discuss the status of the B-factory benchmark modes in view of the current experimental data. We shall then turn to the "El Dorado" for B-decay studies at hadron colliders, the  $B_s$ -meson system, where we will also address new, theoretically clean strategies to explore CP violation.

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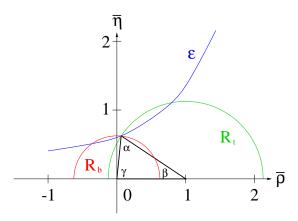
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## 1 Setting the Stage

#### 1.1 Preliminaries

The discovery of CP violation in 1964 through the observation of  $K_L \to \pi^+\pi^-$  decays came as a big surprise [1]. As is well known, this particular manifestation of CP violation, which is described by the famous parameter  $\varepsilon_K$ , originates from the fact that the mass eigenstate  $K_L$  is not a pure CP eigenstate with eigenvalue -1, but one that receives also a tiny admixture of the CP-even eigenstate. After tremendous efforts, also "direct" CP violation, i.e. CP-violating effects arising directly at the decay-amplitude level, could be established by the NA48 (CERN) and KTeV (FNAL) collaborations in 1999, through a measurement of a non-vanishing value of  $\operatorname{Re}(\varepsilon_K'/\varepsilon_K)$  [2]. Unfortunately, this observable does not allow a stringent test of the Standard-Model description of CP violation, unless significant theoretical progress concerning the relevant hadronic matrix elements can be made (for a detailed discussion, see [3]).

In this decade, the exploration of CP violation is governed by B mesons, which provide various tests of the Kobayashi-Maskawa mechanism [4], allowing us to accommodate this phenomenon in the Standard Model (SM). Moreover, these studies offer also interesting insights into hadron dynamics. We will hence focus on the B-meson system in the following discussion; a considerably more detailed review can be found in [5]. At the moment, the experimental stage is governed by the asymmetric  $e^+e^-$  B factories operating at the  $\Upsilon(4S)$  resonance, with their detectors BaBar (SLAC) and Belle (KEK). These experiments have already established CP violation in  $B_d \rightarrow J/\psi K_S$  decays in 2001, which represents the beginning of a new era in the exploration of CP violation, and many interesting strategies can now be confronted with the data [6]. In the near future, we expect also interesting B-physics results from run II of the Tevatron, which will provide - among other things – first access to decays of  $B_s$  mesons [7]. In the era of the LHC, these modes can then be fully exploited [8], in particular at LHCb (CERN) and BTeV (FNAL).



**Figure 1**. Contours to determine the UT in the  $\overline{\rho} - \overline{\eta}$  plane.

## 1.2 Central Target: Unitarity Triangle

The main goal is to overconstrain as much as possible the apex of the unitarity triangle (UT) of the Cabibbo–Kobayashi–Maskawa (CKM) matrix in the plane of the generalized Wolfenstein parameters  $\overline{\rho}$  and  $\overline{\eta}$  [ 9]. On the one hand, we may obtain indirect information on the UT angles through the "CKM fits" [ 10], where the following ingredients enter, as illustrated in Fig. 1: using semileptonic B decays caused by  $b \to u\ell \overline{V}_\ell$ ,  $c\ell \overline{V}_\ell$  quark-level transitions, we may determine the side  $R_b \propto |V_{ub}/V_{cb}|$ , whereas  $R_t \propto |V_{td}/V_{cb}|$  can be determined, within the SM, with the help of  $B_q^0 - \overline{B}_q^0$  mixing  $(q \in \{d,s\})$ . Moreover, the SM interpretation of  $\varepsilon_K$  allows us to fix a hyperbola in the  $\overline{\rho} - \overline{\eta}$  plane. Following these lines, we obtain the following typical ranges for the UT angles:

$$70^{\circ} \lesssim \alpha \lesssim 130^{\circ}, \quad 20^{\circ} \lesssim \beta \lesssim 30^{\circ}, \quad 50^{\circ} \lesssim \gamma \lesssim 70^{\circ}. \quad (1)$$

On the other hand, the measurement of CP-violating effects in *B*-meson decays allows us to obtain *direct* information on  $\alpha$ ,  $\beta$  and  $\gamma$ . In this context, non-leptonic transitions play the key rôle. Within the SM, the unitarity of the CKM matrix allows us to write the amplitude for any given non-

leptonic B decay in the following way:

$$A(\overline{B} \to \overline{f}) = e^{+i\varphi_1} |A_1| e^{i\delta_1} + e^{+i\varphi_2} |A_2| e^{i\delta_2}$$
 (2)

$$A(B \to f) = e^{-i\phi_1} |A_1| e^{i\delta_1} + e^{-i\phi_2} |A_2| e^{i\delta_2},$$
 (3)

where the  $\varphi_{1,2}$  are CP-violating weak phases, which are introduced by the CKM matrix, while the CP-conserving strong amplitudes  $|A_{1,2}|e^{i\delta_{1,2}}$  encode the hadron dynamics of the given decay, i.e. QCD is at work in these quantities. Using these amplitude parametrizations yields

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}} \equiv \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})} = \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin(\varphi_1 - \varphi_2)}{|A_1|^2 + 2|A_1||A_2|\cos(\delta_1 - \delta_2)\cos(\varphi_1 - \varphi_2) + |A_2|^2},$$
(4)

which shows nicely that this kind of CP violation – "direct" CP violation – originates from different decay amplitudes with both different weak and different strong phases. If such a CP asymmetry is measured, the goal is to extract the weak phase difference  $\varphi_1 - \varphi_2$ , as it is related to the angles of the UT and is typically given by  $\gamma$ . However, we observe immediately that the strong amplitudes

$$|A|e^{i\delta} \sim \sum_{k} \underbrace{C_{k}(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \overline{f}|Q_{k}(\mu)|\overline{B}\rangle}_{\text{"unknown"}}$$
 (5)

introduce large uncertainties into the game through nonperturbative hadronic matrix elements of local four-quark operators, which are poorly known.

#### 1.3 Main Avenues to Explore CP Violation

In order to tackle this challenging problem, we may follow three main avenues. First, we may try to calculate the hadronic matrix elements entering (5), which is a very challenging issue. Nevertheless, as was discussed at this workshop by C.T. Sachrajda and Z. Wei, interesting progress could recently be made in this direction through the development of the QCD factorization and perturbative hard-scattering formalisms, as well as soft collinear effective theory (for a comprehensive review, see [11]).

In order to convert measurements of CP asymmetries of the kind specified in (4) into solid information on the angles of the UT, it is desirable to reduce as much as possible the theoretical input on hadronic matrix elements. Such strategies, allowing in particular the determination of  $\gamma$ , are provided by fortunate cases, where we may eliminate the hadronic matrix elements through relations between different decay amplitudes: we distinguish between exact relations, involving pure tree-diagram-like decays of the kind  $B \to DK$  or  $B_c \to DD_s$ , and relations, which follow from the flavour symmetries of strong interactions, involving  $B_{(s)} \to \pi\pi, \pi K, KK$  decays (see [5] and references therein).

The third avenue we may follow is to employ decays of neutral  $B_q$  mesons  $(q \in \{d,s\})$ , where we may obtain interference effects between  $B_q^0 - \overline{B_q^0}$  mixing and decay processes, leading to another type of CP violation, "mixing-induced" CP violation, which allows us to play many games. If such a transition is dominated by a single weak amplitude, i.e. the sums in (2) and (3) run only over a single term, the corresponding "unknown" hadronic matrix element cancels in the mixing-induced CP asymmetry; the most important example in this context is the "golden" mode  $B_d \to J/\psi K_{\rm S}$ . It is also very interesting and useful to complement relations between different decay processes with such CP-violating observables.

## 2 The B-Factory Benchmark Modes

#### 2.1 The Amplitude Relation Avenue: $B \rightarrow \pi K$

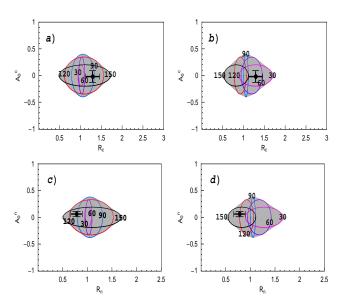
These modes originate from  $\overline{b} \to \overline{d} d\overline{s}, \overline{u}u\overline{s}$  quark-level processes, and may receive contributions both from penguin and from tree topologies, where the latter involve the UT angle  $\gamma$ . Since the ratio of tree to penguin contributions is governed by the tiny CKM factor  $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02$ ,  $B \to \pi K$  decays are dominated by QCD penguins, despite their loop suppression. As far as electroweak (EW) penguins are concerned, their effects are expected to be negligible in the case of the  $B_d^0 \to \pi^- K^+$ ,  $B^+ \to \pi^+ K^0$  system, as they contribute here only in colour-suppressed form. On the other hand, EW penguins may also contribute in colourallowed form to  $B^+ \to \pi^0 K^+$  and  $B_d^0 \to \pi^0 K^0$ , and are hence expected to be sizeable in these modes, i.e. of the same order of magnitude as the tree topologies.

Thanks to interference effects between tree and penguin amplitudes, we obtain a sensitivity on  $\gamma$ . In order to determine this angle, we may use an isospin relation as a starting point, suggesting the following combinations: the "mixed"  $B^{\pm} \to \pi^{\pm} K$ ,  $B_d \to \pi^{\mp} K^{\pm}$  system [12]–[15], the "charged"  $B^{\pm} \to \pi^{\pm} K$ ,  $B^{\pm} \to \pi^0 K^{\pm}$  system [16]–[18], and the "neutral"  $B_d \to \pi^0 K$ ,  $B_d \to \pi^{\mp} K^{\pm}$  systems can be described by the same set of formulae, just making straightforward replacements of variables. Let us first focus on the charged and neutral  $B \to \pi K$  systems. In order to determine  $\gamma$  and the corresponding strong phases, we have to introduce appropriate CP-conserving and CP-violating observables:

$$\frac{R_{\rm c}}{A_0^{\rm c}} \equiv 2 \left[ \frac{{\rm BR}(B^+ \to \pi^0 K^+) \pm {\rm BR}(B^- \to \pi^0 K^-)}{{\rm BR}(B^+ \to \pi^+ K^0) + {\rm BR}(B^- \to \pi^- \overline{K^0})} \right]$$
(6)

$$\begin{array}{ll} R_{\rm n} & \equiv \frac{1}{2} \left[ \frac{{\rm BR}(B_d^0 \to \pi^- K^+) \pm {\rm BR}(\overline{B_d^0} \to \pi^+ K^-)}{{\rm BR}(B_d^0 \to \pi^0 K^0) + {\rm BR}(\overline{B_d^0} \to \pi^0 \overline{K^0})} \right], \ (7) \end{array}$$

where the  $R_{c,n}$  and  $A_0^{c,n}$  refer to the plus and minus signs, respectively. For the parametrization of these observables,



**Figure 2.** The allowed regions in observable space of the charged  $(r_c = 0.20; (a), (b))$  and neutral  $(r_n = 0.19; (c), (d))$   $B \to \pi K$  systems for q = 0.68: in (a) and (c), we show also the contours for fixed values of  $\gamma$ , whereas we give the curves arising for fixed values of  $|\delta_c|$  and  $|\delta_n|$  in (b) and (d), respectively.

we employ the isospin relation mentioned above, and assume that certain rescattering effects are small, which is in accordance with the QCD factorization picture [20]; large rescattering processes would be indicated by  $B \rightarrow KK$  modes, which are already strongly constrained by the B factories, and could be included through more elaborate strategies [15, 17, 18]. Following these lines, we may write

$$R_{c,n} = fct(q, r_{c,n}, \delta_{c,n}, \gamma), \quad A_0^{c,n} = fct(r_{c,n}, \delta_{c,n}, \gamma),$$
(8)

where the parameters q,  $r_{c,n}$  and  $\delta_{c,n}$  have the following meaning: q describes the ratio of EW penguin to tree contributions, and can be determined with the help of SU(3)flavour-symmetry arguments, yielding  $q \sim 0.7$  [ 16]. On the other hand,  $r_{c,n}$  measures the ratio of tree to QCD penguin topologies, and can be fixed through SU(3) arguments and data on  $B^{\pm} \to \pi^{\pm} \pi^{0}$  modes [21], which give  $r_{\rm c,n} \sim 0.2$ . Finally,  $\delta_{c,n}$  is the CP-conserving strong phase between the tree and QCD penguin amplitudes. Since we may fix q and  $r_{c,n}$ , the observables  $R_{c,n}$  and  $A_0^{c,n}$  actually depend only on the two "unknown" parameters  $\delta_{c,n}$  and  $\gamma$ . If we vary them within their allowed ranges, i.e.  $-180^{\circ} \le \delta_{c,n} \le +180^{\circ}$  and  $0^{\circ} \le \gamma \le 180^{\circ}$ , we obtain an allowed region in the  $R_{\rm c,n}$ – $A_0^{\rm c,n}$ plane [22, 23]. Should the measured values of  $R_{c,n}$  and  $A_0^{c,n}$  fall outside this region, we would have an immediate signal for new physics (NP). On the other hand, should the measurements lie inside the allowed range,  $\gamma$  and  $\delta_{c,n}$ could be extracted. The value of  $\gamma$  thus obtained could then be compared with the results of other strategies, whereas the strong phase  $\delta_{c,n}$  would offer interesting insights into

hadron dynamics. This exercise can be performed separately for the charged and neutral  $B \to \pi K$  systems.

In Fig. 2, we show the allowed regions in the  $R_{c,n}$ - $A_0^{c,n}$ planes [23], where the crosses represent the averages of the current B-factory data. As can be read off from the contours in these figures, both the charged and the neutral  $B \to \pi K$  data favour  $\gamma \gtrsim 90^{\circ}$ , which would be in conflict with the  $\gamma$  range in (1) following from the "standard analysis" of the UT. Interestingly, the charged modes point towards  $|\delta_c| \leq 90^\circ$  (factorization predicts  $\delta_c$  to be close to  $0^{\circ}$  [ 24]), whereas the neutral decays seem to prefer  $|\delta_{\rm n}| \gtrsim 90^{\circ}$ . Since we do not expect  $\delta_{\rm c}$  to differ significantly from  $\delta_n$ , we arrive at a "puzzling" picture of the kind that was already considered a couple of years ago in [19]. On the other hand, the data for the mixed  $B \to \pi K$  system fall well into the SM region in observable space and do not indicate any "anomalous" behaviour. A detailed discussion of this " $B \rightarrow \pi K$  puzzle", which may be a manifestation of new physics in the EW penguin sector, and its relation to rare B and K decays, was recently given in [25]. It will be very exciting to follow the evolution of the data.

#### 2.2 The Neutral *B*-Decay Avenue

### 2.2.1 Time-Dependent CP Asymmetries

A particularly simple but very important special case arises for neutral  $B_q$ -meson decays  $(q \in \{d,s\})$  into final CP eigenstates  $|f\rangle$ , which satisfy  $(\mathscr{CP})|f\rangle = \pm |f\rangle$ . Here we obtain the following expression [5]:

$$\frac{\Gamma(B_q^0(t) \to f) - \Gamma(\overline{B_q^0}(t) \to \overline{f})}{\Gamma(B_q^0(t) \to f) + \Gamma(\overline{B_q^0}(t) \to \overline{f})}$$

$$= \begin{bmatrix} \mathscr{A}_{CP}^{dir} \cos(\Delta M_q t) + \mathscr{A}_{CP}^{mix} \sin(\Delta M_q t) \\ \cosh(\Delta \Gamma_q t/2) - \mathscr{A}_{\Lambda \Gamma} \sinh(\Delta \Gamma_q t/2) \end{bmatrix}, \tag{9}$$

where

$$\mathscr{A}_{\text{CP}}^{\text{dir}} \equiv \frac{1 - \left| \xi_f^{(q)} \right|^2}{1 + \left| \xi_f^{(q)} \right|^2} \quad \text{and} \quad \mathscr{A}_{\text{CP}}^{\text{mix}} \equiv \frac{2 \operatorname{Im} \xi_f^{(q)}}{1 + \left| \xi_f^{(q)} \right|^2}, \tag{10}$$

with

$$\xi_f^{(q)} = \mp e^{-i\phi_q} \left[ \frac{A(\overline{B_q^0} \to \overline{f})}{A(B_q^0 \to f)} \right],\tag{11}$$

describe the "direct" and "mixing-induced" CP-violating observables, respectively. In the SM, the CP-violating weak  $B_q^0 - \overline{B_q^0}$  mixing phase  $\phi_q$  is associated with the well-known box diagrams, and is given by

$$\phi_{q} = 2\arg(V_{tq}^{*}V_{tb}) = \begin{cases} +2\beta & (q=d) \\ -2\lambda^{2}\eta & (q=s), \end{cases}$$
 (12)

where  $\beta$  is the usual angle of the UT. Looking at (9), we observe that  $\Delta\Gamma_q$  provides another observable  $\mathscr{A}_{\Delta\Gamma}$ , which is, however, not independent from those in (10).

# **2.2.2** $B_d \rightarrow J/\psi K_S$

One of the most famous *B*-meson decays, the "golden" mode  $B_d^0 \to J/\psi K_{\rm S}$  to extract  $\sin 2\beta$  [26], originates from  $\overline{b} \to \overline{c} c \overline{s}$  quark-level processes. Within the SM, it receives contributions both from tree and from penguin topologies, so that we may write the decay amplitude as follows:

$$A(B_d^0 \to J/\psi K_{\rm S}) \propto \left[1 + \lambda^2 a e^{i\theta} e^{i\gamma}\right], \eqno(13)$$

where the hadronic parameter  $ae^{i\theta}$  is a measure of the ratio of the penguin to tree contributions [27]. Since this quantity enters in a doubly Cabibbo-suppressed way, and is naïvely expected to be of  $\mathcal{O}(\overline{\lambda})$ , where  $\overline{\lambda} = \mathcal{O}(\lambda) = \mathcal{O}(0.2)$  is a "generic" expansion parameter [28], we arrive at

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to J/\psi K_{\mathrm{S}}) = 0 + \mathscr{O}(\overline{\lambda}^3)$$
 (14)

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{S}}) = -\sin\phi_d + \mathscr{O}(\overline{\lambda}^3). \tag{15}$$

The decay  $B_d^0 \to J/\psi K_{\rm S}$  and similar channels led to the observation of CP violation in the *B* system in 2001 [6]; the current status of  $\sin \phi_d \stackrel{\rm SM}{=} \sin 2\beta$  is given as follows:

$$\sin 2\beta = \begin{cases} 0.741 \pm 0.067 \pm 0.033 & (BaBar [29]) \\ 0.733 \pm 0.057 \pm 0.028 & (Belle [30]), \end{cases}$$
 (16)

yielding the world average

$$\sin 2\beta = 0.736 \pm 0.049,\tag{17}$$

which agrees well with the results of the "CKM fits" of the UT summarized in (1), implying  $0.6 \lesssim \sin 2\beta \lesssim 0.9$ .

In the LHC era [8], the experimental accuracy of  $\sin 2\beta$  may reach a level requiring deeper insights into the corrections affecting (15). A possibility to control them is provided by the  $B_s \to J/\psi K_S$  channel [27]. Moreover, also direct CP violation in  $B_d \to J/\psi K_S$  allows us to probe these effects. So far, there are no experimental indications for a non-vanishing value of  $\mathscr{A}_{\mathrm{CP}}^{\mathrm{cir}}(B_d \to J/\psi K_S)$ .

The agreement between (17) and the "CKM fits" is striking. However, it should not be forgotten that NP may nevertheless hide in  $\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{S}})$ . The point is that the key quantity is actually  $\phi_d$  itself, which is given by

$$\phi_d = (47 \pm 4)^\circ \lor (133 \pm 4)^\circ. \tag{18}$$

Here the former value agrees perfectly with  $40^{\circ} \lesssim 2\beta \stackrel{\text{SM}}{=} \phi_d \lesssim 60^{\circ}$ , which is implied by the "CKM fits", whereas the latter would correspond to NP. The two solutions can obviously be distinguished through a measurement of the sign

of  $\cos \phi_d$ . To accomplish this important task, several strategies were proposed [31], but their practical implementations are unfortunately rather challenging. One of the most accessible approaches employs the time-dependent angular distribution of the  $B_d \to J/\psi[\to \ell^+\ell^-]K^*[\to \pi^0K_S]$  decay products, allowing us to extract  $\operatorname{sgn}(\cos \phi_d)$  if we fix the sign of a hadronic parameter  $\cos \delta_f$ , which involves a strong phase  $\delta_f$ , through factorization [32, 33].

# **2.2.3** $B_d \rightarrow \phi K_S$

Another important testing ground for the SM description of CP violation is provided by the decay  $B_d \to \phi K_S$ , which originates from  $\overline{b} \to \overline{s}s\overline{s}$  quark-level processes. In analogy to its charged counterpart  $B^{\pm} \to \phi K^{\pm}$ , this mode is governed by QCD penguins [34], but also EW penguin contributions are sizeable [35, 36]. Since such penguin topologies are absent at the tree level in the SM,  $B \to \phi K$  decays represent a sensitive probe for NP effects. Within the SM, we obtain the following relations [37]–[41]:

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \phi K_{\mathrm{S}}) = 0 + \mathscr{O}(\overline{\lambda}^2)$$
 (19)

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \phi K_{\mathrm{S}}) = \mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{S}}) + \mathscr{O}(\overline{\lambda}^2).$$
 (20)

The current experimental status of the CP-violating  $B_d \rightarrow \phi K_S$  observables is given as follows [42, 43]:

$$\mathscr{A}_{CP}^{dir} = \begin{cases} -0.38 \pm 0.37 \pm 0.12 & (BaBar) \\ +0.15 \pm 0.29 \pm 0.07 & (Belle) \end{cases}$$
 (21)

$$\mathscr{A}_{CP}^{mix} = \begin{cases} -0.45 \pm 0.43 \pm 0.07 & (BaBar) \\ +0.96 \pm 0.50^{+0.11}_{-0.09} & (Belle). \end{cases}$$
 (22)

Since we have, on the other hand,  $\mathcal{M}_{CP}^{mix}(B_d \to J/\psi K_S) = -0.736 \pm 0.049$ , we arrive at a puzzling situation, which has already stimulated many speculations about NP effects in  $B_d \to \phi K_S$  [44]. However, because of the large experimental uncertainties and the unsatisfactory current situation, it seems too early to get too excited by the possibility of having large NP contributions to the  $B_d \to \phi K_S$  decay amplitude. It will be very interesting to observe how the B-factory data will evolve, and to keep an eye on  $B_d \to \eta' K_S$  and other related modes.

# **2.2.4** $B_d \to \pi^+\pi^-$

The  $B_d^0 \to \pi^+\pi^-$  channel is another prominent *B*-meson transition, originating from  $\overline{b} \to \overline{u}u\overline{d}$  quark-level processes. In the SM, we may write

$$A(B_d^0 \to \pi^+ \pi^-) \propto \left[ e^{i\gamma} - de^{i\theta} \right],$$
 (23)

where the CP-conserving strong parameter  $de^{i\theta}$  measures the ratio of the penguin to tree contributions [45]. In contrast to the  $B_d^0 \to J/\psi K_{\rm S}$  amplitude (13), this parameter

does *not* enter (23) in a doubly Cabibbo-suppressed way, thereby leading to the well-known "penguin problem" in  $B_d \to \pi^+\pi^-$ . If we had negligible penguin contributions, i.e. d=0, things would simplify as follows:

$$\mathscr{A}_{CP}^{\text{dir}}(B_d \to \pi^+ \pi^-) = 0 \tag{24}$$

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+ \pi^-) = \sin(\phi_d + 2\gamma) \stackrel{\mathrm{SM}}{=} -\sin 2\alpha, \quad (25)$$

where we have used  $\phi_d \stackrel{\text{SM}}{=} 2\beta$  and the unitarity relation  $2\beta + 2\gamma = 2\pi - 2\alpha$  in the last identity. We observe that actually  $\phi_d$  and  $\gamma$  enter directly  $\mathscr{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)$ , and not  $\alpha$ . Consequently, since  $\phi_d$  can straightforwardly be fixed through  $B_d \to J/\psi K_{\text{S}}$ , we may use  $B_d \to \pi^+\pi^-$  to probe  $\gamma$ . The current status of the CP-violating  $B_d \to \pi^+\pi^-$  observables is given as follows:

$$\mathscr{A}_{CP}^{dir} = \begin{cases} -0.19 \pm 0.19 \pm 0.05 & (BaBar [46]) \\ -0.77 \pm 0.27 \pm 0.08 & (Belle [47]) \end{cases}$$
 (26)

$$\mathscr{A}_{CP}^{mix} = \begin{cases} +0.40 \pm 0.22 \pm 0.03 & (BaBar [ 46]) \\ +1.23 \pm 0.41^{+0.07}_{-0.08} & (Belle [ 47]). \end{cases}$$
 (27)

The BaBar and Belle results are not fully consistent with each other. If we calculate, nevertheless, the weighted averages of (26) and (27), we obtain

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+ \pi^-) = -0.39 \pm 0.16 \, (0.27)$$
 (28)

$$\mathscr{A}_{CP}^{mix}(B_d \to \pi^+ \pi^-) = +0.58 \pm 0.19 (0.34),$$
 (29)

where the errors in brackets are those increased by the PDG scaling-factor procedure [48]. Should large direct CP violation in  $B_d \to \pi^+\pi^-$ , as suggested by (28), be confirmed by future data, we would require large penguin contributions with large CP-conserving strong phases. A significant impact of penguins on  $B_d \to \pi^+\pi^-$  is also indicated by the data on the  $B \to \pi K, \pi\pi$  branching ratios [23, 49], as well as by theoretical considerations [24, 50, 51]. Consequently, it is already evident that we *must* take the penguin contributions to  $B_d \to \pi^+\pi^-$  into account.

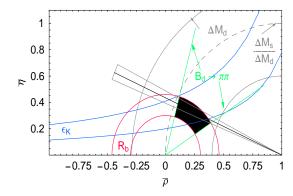
A possibility to deal with this problem is provided by the strategies proposed in [23, 49], employing  $B_d \to \pi^{\mp} K^{\pm}$  (for alternative approaches, see [5]). If we make use of SU(3) flavour-symmetry arguments and plausible dynamical assumptions, we may complement

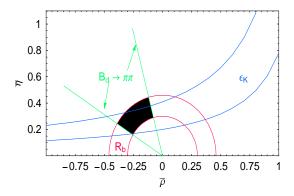
$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+ \pi^-) = \mathrm{fct}(d, \theta, \gamma)$$
 (30)

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+ \pi^-) = \mathrm{fct}(d, \theta, \gamma, \phi_d)$$
 (31)

with

$$H = \left(\frac{1 - \lambda^2}{\lambda^2}\right) \left[\frac{\text{BR}(B_d \to \pi^+ \pi^-)}{\text{BR}(B_d \to \pi^\mp K^\pm)}\right] = \text{fct}(d, \theta, \gamma), \quad (32)$$





**Figure 3.** The allowed regions for the UT fixed through  $R_b$  and CP violation in  $B_d \to \pi^+\pi^-$ , as described in the text: the upper and lower figures correspond to  $\phi_d = 47^\circ$  and  $\phi_d = 133^\circ$ , respectively (H = 7.5).

allowing the extraction of d,  $\theta$  and  $\gamma$ . Taking into account the *B*-factory result  $H \sim 7.5$ , the CP asymmetries in (28) and (29) point towards the following picture: for  $\phi_d \sim 47^\circ$ , we obtain  $\gamma \sim 60^\circ$ , in full accordance with the SM. On the other hand, the unconventional  $\phi_d \sim 133^{\circ}$  solution, which would require CP-violating NP contributions to  $B_d^0 - \overline{B_d^0}$  mixing, favours  $\gamma \sim 120^\circ$ . If we assume a scenario for physics beyond the SM, where we have large NP contributions to  $B_d^0 - B_d^0$  mixing, but not to the  $\Delta B = 1$  and  $\Delta S = 1$  decay processes, which was already considered several years ago [52] and can be motivated by generic arguments and within supersymmetry [53], we may complement  $R_b$  (determined from semileptonic tree decays) with the range for  $\gamma$  extracted from our  $B_d \to \pi^+\pi^-$  analysis, allowing us to fix the apex of the UT in the  $\overline{\rho} - \overline{\eta}$  plane. The results of this exercise are summarized in Fig. 3, following [53], where also ranges for  $\alpha$ ,  $\beta$  and  $\gamma$  are given and a detailed discussion of the theoretical uncertainties can be found. Interestingly, the measured branching ratio for the rare kaon decay  $K^+ \to \pi^+ \nu \overline{\nu}$  seems to point towards  $\gamma > 90^{\circ}$  [ 54], thereby favouring the unconventional solution of  $\phi_d = 133^{\circ}$  [53]. Further valuable information on this exciting possibility can be obtained from the rare decays  $B_{s,d} \to \mu^+ \mu^-$ . We shall return to this issue in Subsection 3.2, discussing the  $B_d \to \pi^+\pi^-$ ,  $B_s \to K^+K^-$  system.

### 3 The "El Dorado" for Hadron Colliders

At the  $e^+e^-$  B factories operating at  $\Upsilon(4S)$ ,  $B_s$  mesons are not accessible. On the other hand, plenty of  $B_s$  mesons will be produced at hadron colliders. Consequently, these particles are the "El Dorado" for B-decay studies at run II of the Tevatron [7], and later on at the LHC [8].

An important aspect of  $B_s$  physics is the mass difference  $\Delta M_s$  of the  $B_s$  mass eigenstates, which can be complemented with  $\Delta M_d$  to determine the side  $R_t \propto |V_{td}/V_{cb}|$  of the UT. To this end, we use that  $|V_{cb}| = |V_{ts}|$  to a good accuracy in the SM, and require just a single SU(3)-breaking parameter, which can be determined, e.g. on the lattice. At the moment, only experimental lower bounds on  $\Delta M_s$  are available, which can be converted into upper bounds on  $R_t$ , implying  $\gamma \lesssim 90^\circ$  [10]. Once  $\Delta M_s$  is measured, more stringent constraints on  $\gamma$  will emerge.

Another interesting quantity is  $\Delta\Gamma_s$ . While  $\Delta\Gamma_d/\Gamma_d$  is negligibly small,  $\Delta\Gamma_s/\Gamma_s$  may be as large as  $\mathcal{O}(10\%)$  (for a recent study, see [55]), thereby allowing interesting CP studies with "untagged"  $B_s$  decay rates, where we do not distinguish between initially present  $B_s^0$  mesons [56].

## 3.1 $B_s \rightarrow J/\psi \phi$

This promising channel is the  $B_s$ -meson counterpart of the "golden" mode  $B_d \to J/\psi K_S$ , and is described by a transition amplitude with a completely analogous structure. In contrast to  $B_d \to J/\psi K_S$ , the final state of  $B_s \to J/\psi \phi$  is an admixture of different CP eigenstates, which can, however, be disentangled through an angular analysis of the  $J/\psi [\to \ell^+ \ell^-] \phi [\to K^+ K^-]$  decay products [57, 58]. Their angular distribution exhibits tiny direct CP violation, whereas mixing-induced CP-violating effects allow the extraction of

$$\sin \phi_s + \mathcal{O}(\overline{\lambda}^3) = \sin \phi_s + \mathcal{O}(10^{-3}). \tag{33}$$

Since we have  $\phi_s = -2\lambda^2 \eta = \mathcal{O}(10^{-2})$  in the SM, the determination of this phase from (33) is affected by generic hadronic uncertainties of  $\mathcal{O}(10\%)$ , which may become an important issue for the LHC era. These uncertainties can be controlled with the help of flavour-symmetry arguments through the decay  $B_d \to J/\psi \rho^0$  [59]. Needless to note, the big hope is that experiments will find a *sizeable* value of  $\sin \phi_s$ , which would immediately signal the presence of NP contributions to  $B_s^0 - \overline{B_s^0}$  mixing.

Other interesting aspects of the  $B_s \to J/\psi \phi$  angular distribution are the determination of the width difference  $\Delta \Gamma_s$  from untagged data samples [58] (for recent feasibility studies for the LHC, see [60]), and the extraction of

 $\cos \delta_f \cos \phi_s$  terms, where the  $\delta_f$  are CP-conserving strong phases. If we fix the signs of  $\cos \delta_f$  through factorization, we may fix the sign of  $\cos \phi_s$ , which allows an *unambiguous* determination of  $\phi_s$  [33]. In this context,  $B_s \to D_{\pm} \eta^{(\prime)}$ ,  $D_{\pm} \phi$ , ... decays are also interesting [61, 62].

3.2 
$$B_s \to K^+K^-$$

The decay  $B_s \to K^+K^-$  is dominated by QCD penguins and complements  $B_d \to \pi^+\pi^-$  nicely, thereby allowing a determination of  $\gamma$  with the help of U-spin flavour-symmetry arguments [45]. Within the SM, we may write

$$A(B_s^0 \to K^+ K^-) \propto \left[ e^{i\gamma} + \left( \frac{1 - \lambda^2}{\lambda^2} \right) d' e^{i\theta'} \right],$$
 (34)

where the hadronic parameter  $d'e^{i\theta'}$  is the  $B_s^0 \to K^+K^-$  counterpart of the  $B_d^0 \to \pi^+\pi^-$  parameter  $de^{i\theta}$  introduced in (23). In analogy to (30) and (31), we then have

$$\mathscr{A}_{CP}^{dir}(B_s \to K^+K^-) = fct(d', \theta', \gamma)$$
 (35)

$$\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to K^+ K^-) = \mathrm{fct}(d', \theta', \gamma, \phi_s). \tag{36}$$

As we saw above,  $\phi_d$  and  $\phi_s$  can "straightforwardly" be fixed, also if NP should contribute to  $B_q^0 - \overline{B_q^0}$  mixing. Consequently,  $\mathscr{A}_{CP}^{dir}(B_d \to \pi^+\pi^-)$  and  $\mathscr{A}_{CP}^{mix}(B_d \to \pi^+\pi^-)$  allow us to eliminate  $\theta$ , thereby yielding d as a function of  $\gamma$  in a *theoretically clean* way. Analogously, we may fix d' as a function of  $\gamma$  with the help of  $\mathscr{A}_{CP}^{dir}(B_s \to K^+K^-)$  and  $\mathscr{A}_{CP}^{mix}(B_s \to K^+K^-)$ .

If we look at the corresponding Feynman diagrams, we observe that  $B_d \to \pi^+\pi^-$  and  $B_s \to K^+K^-$  are related to each other through an interchange of all down and strange quarks. Because of this feature, the *U*-spin flavour symmetry of strong interactions implies

$$d = d', \quad \theta = \theta'. \tag{37}$$

Applying the former relation, we may extract  $\gamma$  and d from the theoretically clean  $\gamma$ –d and  $\gamma$ –d' contours. Moreover, we may also determine  $\theta$  and  $\theta'$ , allowing an interesting check of the second U-spin relation.

This strategy is very promising from an experimental point of view, since experimental accuracies for  $\gamma$  of  $\mathcal{O}(10^\circ)$  and  $\mathcal{O}(1^\circ)$  may be achieved at CDF-II and LHCb, respectively [7, 8, 63]. As far as U-spin-breaking corrections are concerned, they enter the determination of  $\gamma$  through a relative shift of the  $\gamma$ -d and  $\gamma$ -d' contours; their impact on the extracted value of  $\gamma$  depends on the form of these curves, which is fixed through the measured observables. In the examples discussed in [5, 45], the result for  $\gamma$  would be very robust under such corrections. For a more detailed discussion of U-spin-breaking effects and recent attempts to estimate them, the reader is referred to [45, 64, 65].

Interestingly, the quantity H introduced in (32) implies a very narrow SM "target range" in the  $\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to K^+K^-) - \mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-)$  plane [23]. A first important step to complement the analysis discussed in 2.2.4 is the measurement of BR( $B_s \to K^+K^-$ ), which is expected to be available soon from CDF-II. Once also the CP asymmetries of this channel have been measured, we may fully exploit the physics potential of the  $B_s \to K^+K^-$ ,  $B_d \to \pi^+\pi^-$  system as discussed above [45].

**3.3** 
$$B_s \to D_s^{(*)\pm} K^{\mp}$$

Let us finally turn to colour-allowed "tree" decays of the kind  $B_s \to D_s^{(*)\pm} K^{\mp}$ , which have the interesting feature that both a  $B_s^0$  and a  $\overline{B_s^0}$  meson may decay into the same final state, thereby leading to interference between  $B_s^0$  $B_s^0$  mixing and decay processes, which involve the weak phase  $\phi_s + \gamma$  [ 66]. A similar feature is also exhibited by  $B_d \to D^{(*)\pm} \pi^{\mp}$  modes, allowing us to probe  $\phi_d + \gamma$ [ 67]. Whereas the interference effects are governed by  $x_s e^{i\delta_s} \propto R_b \approx 0.4$  in the  $B_s$ -meson case and are hence favourably large, in the  $B_d$  case they are described by  $x_d e^{i\delta_d} \propto -\lambda^2 R_b \approx -0.02$  and hence are tiny. These  $B_s$  and  $B_d$  modes can be treated on the same theoretical basis, and provide new strategies to determine  $\gamma$  [ 68]. To this end, we may write them as  $B_q \to D_q \overline{u}_q$ , where  $D_s \in \{D_s^+, D_s^{*+}, ...\}$ ,  $u_s \in \{K^+, K^{*+}, ...\}$  for q = s, and  $D_d \in \{D^+, D^{*+}, ...\}$ ,  $u_d \in \{\pi^+, \rho^+, ...\}$  for q = d. We shall only consider  $B_q \to D_q \overline{u}_q$  modes, where at least one of the  $D_q$ ,  $\overline{u}_q$  states is a pseudoscalar meson; otherwise a complicated angular analysis has to be performed.

In the "conventional" approach [ 66, 67], the observables of the  $\cos(\Delta M_q t)$  pieces of the time-dependent rate asymmetries are used to extract the parameters  $x_q$ . To this end,  $\mathcal{O}(x_q^2)$  terms have to be resolved. In the case of q = s, we have  $x_s = \mathcal{O}(R_b)$ , implying  $x_s^2 = \mathcal{O}(0.16)$ , so that this may actually be possible, although challenging. On the other hand,  $x_d = \mathcal{O}(-\lambda^2 R_b)$  is doubly Cabibbosuppressed. Although it should be possible to resolve terms of  $\mathcal{O}(x_d)$ , this will be impossible for the vanishingly small  $x_d^2 = \mathcal{O}(0.0004)$  terms, so that other approaches to fix  $x_d$ are required [67]. In order to extract  $\phi_q + \gamma$ , the mixinginduced observables provided by the  $\sin(\Delta M_a t)$  terms have to be measured. Following these lines, we arrive eventually at an eightfold solution for  $\phi_q + \gamma$ . If we fix the sign of  $\cos \delta_a$  with the help of factorization, a fourfold discrete ambiguity emerges. In particular, we may also extract the sign of  $\sin(\phi_q + \gamma)$ , which allows us to distinguish between the two solutions in Fig. 3. In these considerations, the angular momentum L of the  $D_q \overline{u}_q$  state has to be properly taken into account [68].

Let us now discuss other new features of the  $B_q \to D_q \overline{u}_q$  modes, following [68]. As we noted above,  $\Delta \Gamma_s$  may provide interesting "untagged" observables. If we combine them with the "tagged" mixing-induced observables

provided by the  $\sin(\Delta M_s t)$  terms, we may extract, in a simple manner,  $tan(\phi_s + \gamma)$ , which gives an *unambiguous* value for  $\phi_s + \gamma$  itself if we fix again the sign of  $\cos \delta_s$ through factorization. Another important advantage of this new strategy is that only observables proportional to  $\mathcal{O}(x_s)$ are employed, i.e. no  $x_s^2$  terms have to be resolved. Another interesting feature of the  $B_q \to D_q \overline{u}_q$  system is that we may obtain bounds on  $\phi_q + \gamma$ , which may be highly complementary for the  $B_s$  and  $B_d$  modes, thereby implying particularly narrow, theoretically clean ranges for  $\gamma$ . Whereas the  $B_s$  decays are not yet accessible, first results for the  $B_d \to D^{(*)\pm} \pi^{\mp}$  modes obtained by BaBar give  $|\sin(\phi_d + \gamma)| > 0.87$  (68% C.L.) and  $|\sin(\phi_d + \gamma)| > 0.58$ (95% C.L.) [69]. The analysis of these channels at Belle is also in progress [70]. If we look at the  $B_s^0 \to D_s^{(*)+} K^-$  and  $B_d^0 \to D^{(*)+} \pi^-$  decay topologies, we observe that they are related to each other through an interchange of all down and strange quarks. Consequently, the U-spin flavour symmetry of strong interactions implies relations between the corresponding hadronic parameters, which can be implemented in a variety of ways. Apart from features related to multiple discrete ambiguities, the most important advantage of this strategy with respect to the "conventional" approach is that the experimental resolution of the  $x_a^2$  terms is not required. In particular,  $x_d$  does not have to be fixed, and  $x_s$  may only enter through a  $1+x_s^2$  correction, which can straightforwardly be determined through untagged  $B_s$  rate measurements. In the most refined implementation of this strategy, the measurement of  $x_d/x_s$  would *only* be interesting for the inclusion of U-spin-breaking effects. Moreover, we may obtain interesting insights into hadron dynamics and U-spin-breaking effects.

In order to explore CP violation, the colour-suppressed counterparts of the  $B_q \to D_q \overline{u}_q$  modes are also very interesting. In the case of the  $B_d \to DK_{\rm S(L)}, B_s \to D\eta^{(')}, D\phi, \ldots$  modes, we may extract  $\tan \gamma$  in an elegant and unambiguous manner, whereas  $B_s \to D_\pm K_{\rm S(L)}, B_d \to D_\pm \pi^0, D_\pm \rho^0, \ldots$  modes allow very interesting determinations of  $\phi_q$  with theoretical accuracies one order of magnitude higher than those of the conventional  $B_d \to J/\psi K_S, B_s \to J/\psi \phi$  approaches. In particular,  $\phi_s^{\rm SM} = -2\lambda^2 \eta$  could be determined with only  $\mathcal{O}(1\%)$  uncertainty [61, 62].

### 4 Conclusions and Outlook

Thanks to the B factories, CP violation is now a well established phenomenon in the B system, and many strategies to explore CP violation can be confronted with the data. Although the measurement of  $\sin \phi_d$  through  $B_d \to J/\psi K_S$  agrees with the SM – but leaves a twofold solution for  $\phi_d$  itself – the current B-factory data point towards certain "puzzles", for instance in  $B \to \pi K$  and  $B_d \to \phi K_S$  decays. It will be very exciting to see whether these potential discrepancies with the SM will survive improved measurements.

Another important aspect of the exploration of CP violation is the  $B_s$ -meson system, which will be accessible at run II of the Tevatron and can be fully exploited in the era of the LHC. Certainly a promising future of CP violation and quark-flavour physics is ahead of us!

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